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COGGESHAL'S SLIDING Rule.

The description and use of the Sliding rule.

This Rule has four lines upon it, adapted to measuring; marked *A, B, C, D*. Three of these, *A, B, C*, are double lines, because they proceed from 1 to 10 twice over.

The 4th line *D* is a single line, proceeding from 1 to 10, and is called the Girt line.

Two of those lines *B, C*, are upon the Slider, and the other two *A, D*, upon the Rule.

The three single lines are all exactly alike, ⁸⁶ alike numbered 1, 2, 3, 4, to 1 again, from the beginning to the middle; and again, 1, 2, 3, 4, to 10, from the middle to the end.

The 4th line *D*, is numbered 1, 2, 3, 4, to 10; The number 1 in the single line stands against 1 in the double lines, and the num^r. 10 against 10.

In all the lines the divisions are large at the beginning, and continually decrease to the last.

Numeration.

For the Numeration upon the lines; 1 at the beginning may be counted 1, then the 1 in the middle will be 10, and 10 at the end 100.

If 1 at the beginning be called 10, 1 in the middle is 100, and 10 at the end 1000.

Or if the first 1 be reckoned 100, the second 1 will be 1000, and the last 1 at the 10, will be 10,000; and so on.

And as the grand divisions are numbered or altered, so are all the smaller divisions altered in value accordingly.

The use of the double lines *A* and *B*, is for working the Rule of proportion; and for find the Areas of plain Fig^s. And the use of the Girt line *D*, and the other double line *C*, is for measuring Solids.

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Besides these lines, there are several other lines and scales, as a line of Inches with haloes, quarters, and half quarters.

A line of Inches, each divided into ten parts; this is also a foot divided decimally, and serves to turn Inches into decimal parts of a foot.

The backside of the sliding piece is divided into Inches, haloes, quarters, and half quarters; and serves to measure any length, by sliding it out.

There is also a Table shewing what a Load or 50 Feet of Timber comes to from 6 pence to 24 pence p. foot.

On the other side is four diagonal scales, each foot divided into 12 equal parts or Inches for laying down Buildings &c.

There are upon some of these Rules other Scales and Tables and ^{some} of the above left out.

If 1 on *B* be set to any num.^r on *A*, the two lines will be a Table of Multiplication. For the Num.^r on *A*, will be the several Multipliers, and those on *B* the Multiplicands, and the corresponding Num.^r on *A*, the products.

If the lines *C, D*, be set together 10 against 10; they will make a Table of Squares. The line *D* being the Roots, and *C* the Squares; the Num.^r 10 upon *C* being reckend 100, and 10 upon *D* only 10, for the root of 100 is = 10 &c.

In a Gauger's Rule; the line *D* is number'd from 1 at the begining to 10 at the End; w^{ch} is more commodious for extracting roots.

As the shortest and easiest way of measuring superficies and Solids, is by the Sliding rule; it will be necessary to lay down some Problems to shew its use.

PROB. 1.

To Multiply two Numbers together, as 26 by 7.

As $1 : 7 :: 26 : 26 \times 7 = 182.$

B. A. B. A.

By the Rule.

$1 : 7 :: 26 : 182.$

Ans., If the 3^d term on B runs beyond the end of the line A, seek that 3^d term on the other radius (or part) of the line B; then the product will be found on A, which must be increased 10 times as much.

PROB. 2.

To divide one number by another, as 510 by 12.

As $12 : 1 :: 510 : \frac{510}{12} = 42,5$

A. B. A. B.

By the Rule.

$12 : 1 :: 510 : 42,5$

Ans., If the Dividend runs beyond the end of the line B, diminish it 10 or 100 times, to make it fall on B; then increase the quotient 10 or 100 times respectively.

For $\frac{1}{10}^{th}$ or $\frac{1}{100}^{th}$ &c. part of the Dividend, divided by the same divisor, gives $\frac{1}{10}^{th}$ or $\frac{1}{100}^{th}$ &c. part of the Quotient; $\therefore$ the Quotient x'd by 10 or 100 &c. gives the true Quotient required.

PROB. 3.

To square any Number, as 27.

$$\text{As } 1 : 27 :: 27 : 27 \times 27 = 729$$

$$B. \quad A. \quad B. \quad A.$$

By the Rule. $1 : 27 :: 27 : 729$

Or thus.

Set 1 (or 10) upon D to 1 upon C, then against the Num.^r 27 upon D, is the Square upon C = 729.

To reckon right observe.

That if the 10 upon D be counted 1, 10 or 100 &c. the 1 upon C (against it) must be reckon'd 1, 100 or 10,000 &c. 10th are the Squares of the other.

If you would square 270, reckon the 10 on D to be 100, then 1 on C will be 10,000 (because $10,000 = 100^2$) and against 270 on D, will be found 270^2 on C = 72900.

PROB. 4.

To extract the square root of any number, as 4268.

Set 1 upon C, to 1 (or 10) upon D; then against the number 4268 on C, is the root on D = 65, 3.

To value right, you must suppose the one on C to be some of these squares 1, 100, 10,000 &c. 10th is near the given num.^r and then the root corresponding will be the value of the one on D.

DSI

PROB. 5.

To find a mean proportional between any two numbers, 25 and 64.

By Arithmetic.

Investigation.

Call the mean proportional x , then by the Rule of three $25 : x :: x : 64$, which Equation $\times d$ by 25 gives $\frac{x^2 \times 25}{25} = 64 \times 25$, $\div$ ing out 25 $x^2 = 64 \times 25 \therefore x = \sqrt{64 \times 25}$. 10^{th} gives this Rule.

Multiply the two given Numbers together, and out of the product extract the square root gives the mean proportional sought, thus, $64 \times 25 = 1600$ whose root is 40 the mean req^d. For $25 : 40 :: 40 : 64$.

By the Sliding Rule.

By Prob. 1. as $\begin{matrix} b. & a. & b. & a. \\ 1 & : & 25 & :: & 64 & : & 1600, \end{matrix}$
 Whose root by Prob. 1. is = 40.

PROB. 6.

To find a fourth proportional to three given numbers: suppose 12, 28 and 57.

$$12 : 28 :: 57 : 133.$$

$$B : .A :: B : .A.$$

$$12 : 28 :: 57 : 133.$$

NOTE If one of the middle Num^s runs off the line, take the 10th part of it only, and then take 10 times the 4th proportional found.

The finding a 3^d proportional is the same, the 2^d num^r going twice repeated.

PROB. 7.

If the breadth of a Board be given (8 Inches); to find how much in length will make a foot square.

By Arithmetic as $8 : 1 :: 144 : \frac{144}{8} = 18$ Inches.

B. A. B. A.

By the Rule as $8 : 1 :: 144 : 18$ Inches.

PROB. 8.

To find the Circumference of a Circle, the Diameter being given, as 32.

By Arithmetic, as $7 : 22 :: 32 : 100,6$

B. A. B. A.

By the Rule, as $7 : 22 :: 32 : 100,6$

PROB. 9.

The Circumference given = 100,6 to find the dia.

By Arithmetic, as $22 : 7 :: 100,6 : 32$.

B. A. B. A.

By the Rule, as $22 : 7 :: 100,6 : 32$.

MENSURATION,

By the Sliding Rule.

PROB. 10.

Example 1.

Given the Base of a Triangle = 12,4 Feet and the Perpend^r = 8,3 Feet to find the Area in feet.

By the Pen, As 1 : $\frac{12,4}{2}$:: 8,3 : $\frac{12,4 \times 8,3}{2} = 51,46 = \text{Area.}$

B. A. B. A.

By the Rule, As 1 : $\frac{12,4}{2}$:: 8,3 : 51,46 = area.

Example 2.

Given the Base = $\frac{F. I.}{12} 12'' 08$, and the Perpend^r = $\frac{F. I.}{12} 8'' 8$, to find the Area.

$\frac{1}{2} B = 6,33$ Feet, Perpend^r = 10 $\frac{1}{4}$ Inches, now by the Pen,

as 12×12 : $6,33 \times 12$:: $10\frac{1}{4}$: $\frac{6,33 \times 12 \times 10\frac{1}{4}}{12 \times 12} =$
 $= \frac{6,33 \times 10\frac{1}{4}}{12} = A = \text{Area.} \therefore 12 : 6,33 :: 10\frac{1}{4} : A.$

Because by the Rule of Three.

$\frac{6,33 \times 10\frac{1}{4}}{12} = A = 54,86 \text{ Feet.}$

B. A. B. A.

$\therefore$ By the Rule, as 12 : 6,33 :: 10 $\frac{1}{4}$: 54,8

PROB. 11.

Given the Length and Breadth of a Parallelogram in feet, or feet and Decimal parts to find the Area, in feet.

Example 1.

Given $\left\{ \begin{array}{l} AB = 2,4 \text{ Feet} \\ AD = 8,7 \text{ Feet} \end{array} \right\}$ to find the Area.

As $\begin{array}{c} B. \\ 1 \end{array} : \begin{array}{c} A. \\ 8,7 \end{array} :: \begin{array}{c} B. \\ 2,4 \end{array} : \begin{array}{c} A. \\ 20,8 \end{array}$ Feet the area.

N.B. 3 Quarters of a Foot = ,75. One half = ,5 and one Quarter = ,25.

Example 2.

Given the length of a Parallelogram in feet and Inches, and the Breadth in Inches, to find the content in feet.

F I.

Given the length of a Board, 14' 09, Breadth = 16 Inches, to find the Area in feet.

$\begin{array}{c} B. \\ 12 \end{array} : \begin{array}{c} A. \\ 14,75 \end{array} :: \begin{array}{c} B. \\ 16 \end{array} : \begin{array}{c} A. \\ 19,66 \end{array}$ = Area.

Example 3.

Suppose a Stock of Boards measure 12 Feet 8 Inches in Length, and 10 Feet 4 Inches in Breadth, how many Feet is contained therein.

By the Edge of the Rule 12 feet 8 Inches is nearly = 12,67 feet and 10 feet 4 Inches = 12,4 Inches ::

$$\begin{array}{cccc} B. & A. & B. & A. \\ \text{As } 12 & : 12,67 & :: 12,4 & : 130,8 \text{ Feet the Area.} \end{array}$$

Example 4.

What is the Content of a Pane of Glass 10 Inches 1 Quarter long, and 6 Inches 2 Quarters Broad.

By the Edge of the Rule, $10'' 01 = ,856$ and $6'' 02 = ,51$.

$$\begin{array}{cccc} B. & A. & B. & A. \\ \therefore \text{As } 1 & : ,856 & :: ,51 & : ,46 = \text{Area.} \end{array}$$

2.

$$\begin{array}{cccc} B. & A. & B. & A. \\ \text{or as } 12 & : ,856 & :: 6,5 & : ,462 = \text{Area.} \end{array}$$

What comes this to at $8\frac{1}{2}$ p. Foot.

$$\begin{array}{cccc} B. & A. & B. & A. \\ \text{as } 1 & : 8,5 & :: ,462 & : 3,93 = 4 \text{ nearly.} \end{array}$$

Example 5.

Suppose a window light be 3 Feet 5 Inches long, and 1 Foot 4 Inches broad; how many feet is contained therein, and what comes it to at 1^s. 6^d p. foot.

$$\begin{array}{cccc} B. & A. & B. & A. \\ \text{As } 12 & : 3,42 & :: 16 & : 4,56 = \text{Area in feet.} \end{array}$$

$$\begin{array}{ccccccc} B. & A. & B. & A. & S. & D. \\ \text{Now as } 1 & : 1,5 & :: 4,56 & : 6,84 = 6^{\text{sh}} 09^{\frac{1}{2}}. \end{array}$$

N^{ext} to find the value of any sum^{ts} of Foot is found in Shill^{ings} and decimal parts of a Shilling, the Decimal parts are the same as the decimal parts of a Foot, because 12 Inches make one Foot and 12 pence one Shilling; $\therefore$ to find the value of, & of 1 Shilling look upon the Edge of the Rule for, 00 =, & and against that upon the Inches, you'll find 9^½ and near a Quarter, to ^{sh}is 9^½ and near ^¼ of a penny, or near 1 farthing. and so for any other decimal of a Shilling.

PROB. 12.

Given the Breadth of a Board in Inches to find what length will make a foot Square.

Suppose a Board be 8 Inches broad, how much in length will make a Foot Square.

By Arithmetic, 8) 144 (18 Inches.

$$\begin{array}{cccc} B. & A. & B. & A. \end{array}$$

$$\text{By the Rule, as } 8 : 1 :: 144 : 18.$$

PROB. 13.

How many Feet is there in a Circle whose Diameter is 26 Inches.

$$\begin{array}{cccc} & B. & A. & B. & A. \\ \text{As } 1 & : 26 & :: 26 & : 676 = \text{Square of the dia.} \end{array}$$

$$\text{Then, as } 14 : 11 :: 676 : 530 = \text{Area in Inches.}$$

$$\text{And, as } 144 : 1 :: 530 : 3,7 \text{ Feet the Area.}$$

$$\begin{array}{cccc} I. & F. & I. & F. \\ \text{Orthus, } 26 = 2'' & 2 = 2,16 \end{array}$$

$$\text{Then, as } 1 : 2,16 :: 2,16 : 4,66 = \text{Square of the dia. in feet}$$

$$\text{And, as } 14 : 11 :: 4,66 : 3,66 = \text{Area in feet.}$$

or thus.

$$\text{By Prob. 8. as } 7 : 22 :: 2,16 : 6,75 = \text{Circumference.}$$

$$\text{And, as } 1 : \frac{2,16}{2} = 1,08 :: \frac{6,75}{2} = 3,37 : 3,6 \text{ feet the area.}$$

PROB. 14.

How many Quarries 9 Inches square, will lay a Floor, 16 Feet long, and 14 Feet Broad; and what cometh they to at 3 Pence a piece.

$$\begin{array}{ccc} I. & F. & D. & S. \\ 9 = ,75 & \text{and } 3 = ,25 \end{array}$$

$$\begin{array}{cc} B. & A. & B. & A. \end{array}$$

$$\text{Then, as } 1 : ,75 :: ,75 : ,56 = \text{Area of one quarry.}$$

$$\text{And, as } ,56 : 1 :: 16 \times 14 = 224 : 400 \text{ Quarries.}$$

$$\therefore 1 : ,25 :: 400 : 100^s = 5 L.$$

PROB. 15.

How many Bricks will lay a Brick length wall whose length is 18 Feet and height 11 Feet 9 Inches.

Ans. Workman reckon 50 for a Brick Breadth, 100 for a Brick length, 150 for a Brick and half thick wall &c. for every Yard square.

$$\begin{array}{ccccccc}
 & B. & & A. & & B. & & A. \\
 \therefore & \text{as} & 9 & : & 10 & :: & 11,75 & : & 23,5 \text{ Square yards} \\
 & & & & & & & & 10^{\text{th}} \text{ answers to } 23\frac{1}{2} \text{ hundreds,} = 2350 \text{ Bricks.}
 \end{array}$$

What comes these to at 14.^s p. 1000.

$$\begin{array}{ccccccc}
 & & & S. & & & S. & D. \\
 \text{as} & 1000 & : & 14 & :: & 2350 & : & 32,9^{\text{s}} = 32^{\text{s}} 10\frac{3}{4} = \\
 & & & L. & S. & D. & & \\
 & & & = 1^{\text{L}} 12^{\text{s}} 10\frac{3}{4}.
 \end{array}$$

What comes them to laying at 11.^s p. 1000.

$$\begin{array}{ccccccc}
 & & & S. & & & S. & D. \\
 \text{as} & 1000 & : & 11 & :: & 2350 & : & 21,4^{\text{s}} = 21^{\text{s}} 4\frac{1}{4}.
 \end{array}$$

PROB. 16.

How many Square of tiling is there in the Roof of a House whose length is 18 Feet long, and 12 feet 6 Inches broad.

Ans. $10 \times 10 = 100$ Square feet in a Square of Tiling.

$$\begin{array}{ccccccc}
 & B. & & A. & & B. & & A. \\
 \therefore & \text{as} & 100 & : & 18 & :: & 12,5 & : & 2,25 \text{ Square.}
 \end{array}$$

PROB. 17.

How many Tiles will tile a Roof 18 Feet Long, and 12 feet 6 Inches Broad, allowing 1000 Tiles to $1\frac{1}{2}$ Square: and what come they to at 17^{s. 6d.} p. 1000.

$$\begin{array}{rcllcl}
 & B. & A. & B. & A. \\
 \text{as} & 100 & : 18 & :: 12,5 & : 2,25 \text{ Square.} \\
 \text{Then, as} & 1,5 & : 1000 & :: 2,25 & : 1500 \text{ Tiles.} \\
 \text{And} & 1000 & : 17,5 & :: 1500 & : 26,25 = 1^{\text{s.}} 06^{\text{d.}} 03^{\text{q.}}
 \end{array}$$

PROB. 18.

To measure irregular Parallelograms, such as irregular planks, &c.

RULE.

Take the Breadth in several equidistant places, the sum of 16th divide by the Num^r. of places, for the mean breadth; xply the mean breadth by the length, for the Area.

Example.

Suppose a Plank 16 feet long, and the Breadth in several places 6, 7, 9, 9, 11 Inches.

$$6 + 7 + 9 + 9 + 11 = 42, \text{ and } 5 \overline{) 42} \begin{array}{l} 8 \\ 4 \end{array} = \text{mean breadth.}$$

$$B. \quad A. \quad B. \quad A.$$

$$\text{Then as } 12 : 16 :: 8,4 : 11,2 \text{ feet.}$$

Ans. If the Breadths are taken where they alter, as at 6, 7, 9, 9, 11. If they are not taken at equal distances, the Rule comes near the truth.

SOLIDS.

What is the Solid content of a Cube whose side is 2 Feet 2 Inches.

F. I. I.

F. I. F.

2" 2 = 26, and by the Rule, 2" 2 = 2,166

When the side of the Square of a Piece of Timber can be had, it will always hold.

D.

C.

D.

C.

As 12 : Length in feet, :: Side square in Inches : Content in feet . . ::

D.

C.

D.

C.

As, 12 : 2,16 :: 26 : 10,1 Feet the Solidity.

PROB. 20.

What is the Content of a Square Parallelopipedon whose side is 2 feet 6 Inches and Length 19 Feet.

D.

C.

D.

C.

As 12 : 19 :: 30 : 118,7 Feet.

PROB. 21.

To find the Solidity of any Parallelopipedon, Prism, or Cylinder.

B.

A.

B.

A.

As 1 : Area end in feet and Dec. parts :: Length in feet : Content in feet .

or:

As 144 : Area end in Inches :: Length in feet : Content in feet.

Of Pyramids and Cones.

B

.A.

B.

.A.

As 1 : Area Base in feet and Dec. parts :: $\frac{1}{3}$ height in feet : Content in feet.

or,

As 144 : Area Base in Inches :: $\frac{1}{3}$ height in feet : Content in feet.

Conoids.

The two first terms as before, :: $\frac{1}{2}$ height in feet : Content in feet.

PROB. 22.

Given the Area of the End of any Parallelopipedon, Prism or Cylinder in Inches, as 27, to find how many Inches in length will make a Solid foot.

B.

.A.

B.

.A.

As 27 : 1 :: 1728 : 64 Inchs.

PROB. 23.

The method of measuring wood as it is practised in all Raff yards, taking a Quarter of the Circumference or Girt in the middle, for the side of the square.

To measure irregular Parallelopipedons, such as pieces of Timber, also the Butts of Timber Trees &c.

RULE 1.

Girt them about in the middle wth a string, and measure it, and take a Quarter of the Girt in Inches for the side of the square; which squared and x'd by the length of the Body in feet, and this product ÷'ed by 144, gives the content in feet.

or,

Or.

If the Girt be taken in feet and decimal parts, $\frac{1}{4}^{th}$ of this squar'd, and x'd by the length in feet gives the Solidity in feet.

Example 1.

A piece of wood is 10 feet long, and the Girt round about the middle, is 66 Inches; to find the Content in feet.

By the Pen.

$$\frac{66}{4} = 16\frac{2}{4} = 16\frac{1}{2} = 16,5 \text{ Inches. } \therefore \frac{16,5 \times 16,5 \times 10}{144} = 34,03 \text{ feet the Solidity.}$$

Or.

I. F. I. F.

$$66 = 5'' 6 = 5,5 \text{ and } \frac{5,5}{4} = 1,375 \therefore 1,375 \times 1,375 \times 10 = 34,03 \text{ feet as before.}$$

By the RULE.

D. C. D. C.

$$\text{As, } 12 : 10 :: 16\frac{1}{2} : 34 \text{ feet.}$$

S. D.

What comes it to at 1'' 08 p. foot Solid.

S. D. S.

By the Rule 1'' 08 = 1,666 $\therefore$

B. A. B. A. L. S. D.

$$\text{As, } 1 : 1,666 :: 34 : 56,4 = 2'' 16'' 04\frac{3}{4}.$$

ALGEBRA.

Dec^r. 27. 1774

1.

Algebra is a kind of specious Arithmetic, as we here make use of Species, i. e. Symbols or Letters to represent Quan^s. of every kind, as well known as unknown. And it is customary to represent known Quan^s. by the first Letters of the Alphabet $a, b, c, d, &c.$ and unknown ones by the last Letters, $x, y, z, &c.$

2.

The Terms of an Algebraic Expression are connected under all their various Relations by proper ^{Symbols} Characters, as mentioned in page 32. Book 1. but we are here to consider the two principal Symbols $+$ and $-$ in a more extensive view than we did there, for they are designed to represent any two contrary Modes, Kinds, or Actions, &c. In short, what ever is represented by the affirmative Sign $+$, as $+a$; the Contrary is represented by the negative Sign $-$, as $-a$.

Thus.

If $+a$ signifies any Thing $\left\{ \begin{array}{l} \text{added above} \\ \text{to the Right} \\ \text{forwards,} \end{array} \right\}$ *$-a$ signifies* $\left\{ \begin{array}{l} \text{Subtracted below to the Left,} \\ \text{backwards.} \end{array} \right\}$

If $+$ signifies any $\left\{ \begin{array}{l} \text{Increase,} \\ \text{Gravity,} \\ \text{Money due,} \\ \text{Motion upwards,} \end{array} \right\}$ *$-$ signifies any* $\left\{ \begin{array}{l} \text{Decrease,} \\ \text{Levity,} \\ \text{Money owing,} \\ \text{Motion Downwards.} \end{array} \right\}$

And so on in every kind of Contrariety. And two such Quan^s. connected together in any Case destroy each other's Effect or are equal to Nothing, as $+a - a = 0$. Thus, if a Man has but 10 L. and at the same time owes 10 L. he is worth Nothing.

3.

If the first Quantity or Term be affirmative we neglect the Sign +, as $a+b$ instead of $+a+b$. If an Algebraical Quantity consist of two Terms, it is called a Binomial, as $a+b$; if of three Terms, a Trinomial, as $a+b+c$. If more Terms, a Multinomial. all which are compound Quantities. Simple Quantities consisting of one Term only as $+a$, $+ab$, $+abc$.

4.

When simple Quantities are to be $\times$ d together we do not, generally, use the Symbol $\times$, but place them together without; as ab instead of $a \times b$, and abc for $a \times b \times c$; when compound Quantities are to be represented as $\times$ d, then a Line is drawn over the Factors connected wth the proper Sign $\times$; as $\overline{a+b} \times c$, $\overline{a+x} \times \overline{b-y}$.

5.

When any Num^r. of Fig^{rs}. are prefixed to the Terms of an Algebraic Quantity, they are called Coefficients, and shew how often the Quan^{ty} is to be taken; as $3a$, $5bc$, $2a+5b$, &c. And when no Num^r is prefixed, Unity is always understood, tho' not expressed or written, for the Coefficient; thus a is the same as $1a$, and bc as $1bc$, since anything $\times$ d by 1 is still the same.

6.

Quantities are said to ^{be} like, or similar, that are represented by the same Letter or Letters equally repeated; thus $+3a$ and $-5a$ are like; but a and b , or a and aa are unlike.

What other definitions and Symbols or Characters are used in Algebra, will be explained in their proper places as we go on.

PROB. 1.

To add several Quantities together.

RULE. 1.

If the quantities are like and have like signs; add all the coefficients together, for the coefficient to that quantity, and prefix the same sign.

Examples.

To	+ 5a	To	- 16ab	To	+ 4a - 3x	To	a + 2b - 6ab
add	+ 7a	add	- 5ab	add	+ 5a - x	add	3a + 11b - 4ab
		add	- 2ab	add	+ a - 5x	add	6a + b - ab
	<u>+ 12a</u>		<u>- 23ab</u>		<u>+ 10a - 9x</u>		<u>10a + 11b - 11ab</u>

$$\begin{array}{l}
 4aa - 11bad + 9xy + x - 25y - 9z + 10zx - 40xyx \\
 3aa - 10bad + 10xy + 4x - 4y - 4z + 9zx - 9xyx \\
 9aa - 6bad + xy + 9x - y - 3z + zx - 0xyx
 \end{array}$$

$$16aa - 27bad + 20xy + 14x - 30y - 16z + 20zx - 57xyx$$

RULE. 2.

If like quantities with unlike signs; add all the affirmative coefficients, into one sum; and all the negative ones into another; subtract the lesser sum from the greater, and the difference prefix the sign of the greater, with the proper quantity.

Examples.

To	+ 6a	- 16d	3a - 7b	- 5a - 6d + 9bcd
add	- 3a	+ 3d	- 3a + 0b	+ 0a + 10d - 4bcd
	<u>+ 3a</u>	<u>- 13d</u>	<u>0 + b</u>	<u>+ 3a + 4d + 5bcd</u>

$$\begin{array}{r}
 -125ab \\
 -37ab \\
 +ab \\
 +99ab \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 -162ab \\
 +100ab \\
 -62ab \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 +34x^2y \\
 -8x^2y \\
 -x^2y \\
 +92x^2y \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 -67x^2y \\
 -76x^2y \\
 +126x^2y \\
 \hline
 +50x^2y \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 -2aa - 9bcd + dd + 2e \\
 +7aa - 20bcd - dd + 5e \\
 +3aa + 4bcd \\
 +aa \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 +9aa - 25bcd + 0 + 7e \\
 \hline
 \end{array}$$

RULE 3.

Set down all the unlike quantities with their proper signs.

Examples.

$$\begin{array}{r}
 +2a \\
 +3b \\
 -c \\
 +d \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 +2a + 3b - c + d \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 +13aa + 2a \\
 -4ab - 3a \\
 +bc + 5a \\
 -2dd + 6d \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 +13aa - 4ab + bc - 2dd + 6d \\
 \hline
 \text{Sum.}
 \end{array}$$

$$\begin{array}{r}
 2ee + 3ef - ff + 17 \\
 -3ee + 5ef + 2ff - 11 \\
 +6ee - ef + ab - 3 \\
 -xy - 9ef - 4ab - 10 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 5ee - xy - 2ef + ff - 3ab - 7 \\
 \hline
 \text{Sum.}
 \end{array}$$

$$\begin{array}{r}
 +2abc - 0ab + 0ac + 110 - c \\
 -3abc + 6ab - ac + 17 + d \\
 + bc - 4cd + ad - 100 - 9 \\
 \hline
 -abc + bc - 2ab - 4cd + 7ac + ad + 35 - c + d - 9 \text{ Sum.} \\
 \hline
 \end{array}$$

The reason of addition is evident for like signs; and in unlike signs, it follows from the nature of affirmative and negative quan^s, that the difference ought to be taken, to make up the totals. As if a man owes 10*£*. then 10*£*. ought to be deducted from his stock to find his real worth.

Corollary 1.

When several quan^s are to be added together, it is the same thing, whatever order they are placed. Thus,

$$a + b - c = a - c + b = -c + a + b = b + a - c, \text{ &c. all these are the same.}$$

Corollary 2.

Hence the sum of any num^t. of affirmative quan^s are affirmative; and the sum of any num^t. of negative quan^s is negative.

PROB. 2.

To subtract quantities from one another.

RULE.

Change all the Signs of the Subtrahend, (viz. of those quan^s which are to be subtracted) or suppose them in your mind to be changed. Then add all the quan^s together, as before in addition, and their sum will be the remainder or difference required.

Examples.

From	$+8a$	$16b$	$-11c$	-20	$6a - 3x + 6y - 7$
Take	$+3a$	$-5b$	$+3c$	-6	$0a + 4x + 6y + 5$
Diff.	$+5a$	$+21b$	$-14c$	-14	$-2a - 7x + 0 - 12$

From	$a+b$	$a+b$	$aa+2ab+bb$	$aa-bb$
Take	$a-b$	$-a+b$	$+4ab$	$cc-dd$
Diff.	$+2b$	$+2a$	$aa-2ab+bb$	$aa-bb-cc+dd$

From	$3aa - 2a + cd - dd - ff$	3
Take	$-2aa - 5a - ab - 2dd$	$2a - 2b + 13$
Diff.	$5aa + 3a + cd + ab + dd - ff$	$-2a + 2b - 10$

From	$2a - 3x + 5y - 6$	$4x - 3y + aa - c$
Take	$6a + 4x + 5y + 4$	$3xx + 4yy - ab + d$
Diff.	$-4a - 7x - 0 - 10$	$4x - 3y + aa - c - 3xx - 4yy + ab - d$

Corollary.

Hence, To subtract one quantity from another, is the same thing as to add them together, when all the Signs of the subtrahend are changed.

For it is the same thing to subtract $-$, as to add $+$; and to add $-$, as to subtract $+$.

PROB. 3.

To Multiply one Quantity by another.

RULE.

Multiply every particular term (or simple quantity) of the multiplier, into every term of the Multiplicand, one after another; so that the coefficients be x'd into the coefficients; and the letters into the letters, by placing them altogether, like letters in a word. And prefix + to products of like signs, and - to unlike ones. The sum of all is the product sought.

Examples.

$$\text{Mult. } +a \quad -a \quad +3a \quad -4c \quad a+b$$

$$\text{By } +b \quad -b \quad -2b \quad +5a \quad a+b$$

$$\text{Pro. } +ab \quad +ab \quad -6ab \quad -20cd \quad \begin{array}{l} aa+ab \\ +ab+bb \end{array}$$

$$aa+2ab+bb = \text{Product.}$$

$$\text{Multiply } a+b$$

$$\begin{array}{r} \text{By } a-b \\ \hline aa+ab \\ -ab-bb \\ \hline \end{array}$$

$$\text{Product. } aa+0-bb$$

$$\begin{array}{r} -a-b \\ \hline a-b \\ \hline aa-ab \\ -ab+bb \\ \hline aa-2ab+bb \end{array}$$

$$3a-2b$$

$$5a+4b$$

$$15aa-10ab$$

$$+12ab-8bb$$

$$15aa+2ab-8bb$$

$$aa+ab-bb$$

$$a-b$$

$$a^3+a^2b-ab^2$$

$$-a^2b-ab^2+b^3$$

$$a^3-2ab^2+b^3$$

$$ab - 3cd + rs$$

$$5r - 7d$$

$$5rab - 15r^2d + 5r^2s \\ - 7abd + 21cd^2 - 7rsd$$

$$5rab - 15r^2d + 5r^2s - 7abd + 21cd^2 - 7rsd$$

$$3aa - 2ab + 5$$

$$aa + 2ab - 3$$

$$3a^4 - 2a^3b + 5a^2 \\ + 6a^2b - 4a^2b^2 + 10ab \\ - 9a^2 + 6ab - 15$$

$$3a^4 - 4a^3b - 4a^2 - 4a^2b^2 + 16ab - 15$$

$$aa + bb$$

$$cc - dd$$

$$ccaa + ccbb \\ - ddaa - ddbb$$

$$ccaa + ccbb - ddaa - ddbb$$

$$aa + ab + bb$$

$$a - b + 6$$

$$a^3 + a^2b + ab^2 \\ - a^2b - ab^2 - b^3 \\ + 6a^2 + 6ab + 6bb$$

$$a^3 + 6a^2 - b^3 + 6ab + 6b^2$$

Case 1.

That every term in the Multiplieand must be xed by every term in the Multiplier, is thus made evident.

Let $a+b$ be xed by $c+d$; it is plain, $a+b$ must be taken so often as there are units in c and d , that is, as often as there are units in c , and also as oft as there are units in d . $\therefore$ the product will be $a+b \times c + a+b \times d$. But for the same reason $a+b \times c = ac+bc$, also $a+b \times d = ad+bd$. whence the product will be $ac+bc+ad+bd$; that is, the sum of all the products of every Term xed by every Term.

$$\begin{array}{r} a+b \\ c+d \\ \hline ac+bc \\ +ad+bd \\ \hline ac+bc+ad+bd \end{array}$$

Case 2.

That like signs give $+$, and unlike signs $-$, in the Product will appear thus.

Let $d-a$ be xed by b ; then the product will be bd together with $-a \times b$: But db is too big, as being the product of d by b , instead of $d-a$ by b ($d-a$ being less than d by a); $\therefore db$ being too much, the product $-a \times b$ must be taken from it, and the true product will be $db-ab$; and conseq^{ly} $-ab = -a \times b$.

$$\begin{array}{r} d-a \\ b \\ \hline db-ab \end{array}$$

Case 3.

Let $a-b$ be xed by $c-d$. Then by Case 2^d $a-b \times c = ac-bc$, and by the same, $a-b \times d = ad-bd$; But $ac-bc$ is $a-b$ taken c times, that is, too often by d times $a-b$, $\therefore d$ times $a-b = ad-bd$ must be taken from the product $ac-bc$, or added with the signs changed to make the true product, $ac-bc-ad+bd$.

$$\begin{array}{r} a-b \\ c-d \\ \hline ac-bc \\ -ad+bd \\ \hline ac-bc-ad+bd \end{array}$$

Therefore $+ \times +$ gives $+$; $- \times +$ gives $-$; $+ \times -$ gives $-$; and $- \times -$ gives $+$ in the product.

PROB. 4.

To divide one quantity by another.

RULE. 1.

In simple quan^s m^{ch} will divide without a remainder; divide the number by the number, and put the answer in the quotient. Then throw out all the letters in the dividend m^{ch} are found in the divisor, and place the remaining letters in the quotient. And like signs produce +, and unlike signs —, in the quotient.

Example 1.

$$\begin{array}{r} aa) aab(-b. \\ \underline{aab} \\ 0 \end{array}$$

Example 2.

$$\begin{array}{r} 3ab) 15aabcd(5acd. \\ \underline{15aabcd} \\ 0 \end{array}$$

Example 3.

$$\begin{array}{r} -aa) aab(-b. \\ \underline{aab} \\ 0 \end{array}$$

Ex. 4.

$$\begin{array}{r} -3ab) -15aabcd(+5acd. \\ \underline{-15aabcd} \\ 0 \end{array}$$

Ex. 5.

$$\begin{array}{r} -3ddc) -6ddcc(+2c \\ \underline{-6ddcc} \\ 0 \end{array}$$

Example 6.

$$\begin{array}{r} +6a^2b^2) -10b^3a^3d^3(-3abd^3 \\ \underline{10b^3a^3d^3} \\ 0 \end{array}$$

Example 7.

$$\begin{array}{r} -5a^2b) 10a^2bbd(-2bd \\ \underline{10a^2bbd} \\ 0 \end{array}$$

Ex. 8.

$$\begin{array}{r} 9x^2y) -9x^2y^2b(-yb \\ \underline{-9x^2y^2b} \\ 0 \end{array}$$

Ex. 9.

$$\begin{array}{r} -8xx) -16x^3(+2x \\ \underline{-16x^3} \\ 0 \end{array}$$

RULE 2.

In compound quan^s, range the terms of the divisor and dividend, according to the dimensions of some letter. Then, by Rule 1, divide the first term of the dividend by the first term of the divisor, placing the result in the quotient. Multiply the whole divisor by the quotient, and subtract it from the dividend, to w^{ch} bring down the next term of the dividend, call this the dividend.

Divide the first term of the dividend by the first term of the divisor; then multiply and subtract as before, and repeat the same process till all the quan^s be brought down.

Example 10.

$$a) ab+ac-a(b+c-1 = \text{Quot.}$$

$$\begin{array}{r} ab \\ +ac \\ +ac \\ -a \\ -a \\ \hline 0 \end{array}$$

Example 11.

$$\begin{array}{r} 2b-3c) 2baa-3caa(+aa \\ 2baa-3caa \\ \hline 0 \end{array}$$

Example 12.

$$\begin{array}{r} a+b) ac+bc+ad+bd(c+d \\ ac+bc \\ \hline ad+bd \\ ad+bd \\ \hline 0 \end{array}$$

Example 13.

$$\begin{array}{r} y^2-4) y^3-3yy-4y+12(y-3 \\ y^3 \quad -4y \\ \hline -3yy+12 \\ -3yy+12 \\ \hline 0 \end{array}$$

Example 14.

$$\begin{array}{r} 3a-b) 3a^3-12aa-baa+10ab-2bb(aa-4a+2b. \\ 3a^3 \quad -baa \\ \hline -12aa+10ab \\ -12aa+4ab \\ \hline 6ab-2bb \\ 6ab-2bb \\ \hline 0 \end{array}$$

Example 15.

$$\begin{array}{r} a+b) aa-bb(a-b \\ aa+ab \\ \hline -ab-bb \\ -ab-bb \\ \hline 0 \end{array}$$

RULE 3.

In dividing, when you come to a Remainder of one Term, it is commonly set down with the divisor under it, after the other terms, w^{ch} together make the whole quotient.

Example 16.

$$\begin{array}{r}
 a+x \) \ aa+xx \ (a-x \frac{2xx}{a+x} \\
 \underline{aa+ax} \\
 -ax+xx \\
 \underline{-ax-xx} \\
 2xx
 \end{array}$$

Or, when the divisor does not exactly divide the dividend; place the dividend over the divisor, in form of a fraction; throwing out such letters, as are found in all the terms of both the dividend and divisor.

Example 17.

$$(ax-xx) \ ax+xx \left(\frac{ax+xx}{ax-xx} = \frac{a+x}{a-x} = \text{Quotc.} \right.$$

Example 18.

$$20ad \) \ 10ab+15ac \left(\frac{10ab+15ac}{20ad} = \frac{2b+3c}{4d} = \text{Quotc.} \right.$$

1 +) +(+
2 -) -(+
3 +) -(-
4 -) +(-

Example 19.

$$12ab \) \ 30ax-54ay \left(\frac{30ax-54ay}{12ab} = \frac{5x-9y}{2b} = \text{Quotc.} \right.$$

That like signs give +, and unlike signs -, in the quotient, will appear thus. The divisor xed by the quotient must produce the dividend. $\therefore$ When both are +, the quotient is +, because then $+ \times +$ must produce + in the dividend. When they are both -, the quotient is + again, because $- \times -$ must produce + in the dividend. Again, When the divisor is + and the dividend -, the quotient is -, because $- \times +$ produce - in the dividend. Lastly, If the divisor is -, and the dividend +, the quotient will be -, because $- \times -$ produces + in the dividend.

FRACTIONS.

The Operations of algebraic fractions are exactly the same, as those of vulgar fractions in Arithmetic.

PROB. 5.

To reduce a given quantity to a fraction of a given denominator.

RULE.

Multiply that quantity by the given denominator, and under the product write the same denominator.

Example 1.

Let $a+b$ have the denominator x .

$$\frac{a+b}{x} \times x = \frac{ax+bx}{x} = \text{Fraction req'd.}$$

Example 2.

Let $xx-111$ have the denominator 1.

$$\frac{xx-111}{1} \times 1 = \frac{xx-111}{1} = \text{Fraction red.}$$

Example 3.

Let $\frac{a}{b}$ Have the Denominator $b-c$.

$$\frac{\frac{a}{b} \times b-c}{b-c} = \frac{\frac{ab-ac}{b}}{b-c} = \frac{a-\frac{ac}{b}}{b-c} = \text{Fraction req'd.}$$

Corollary.

The value of a fraction is not altered, by xing both numerator and denominator by the same quantity. Thus, $\frac{rab}{rc} = \frac{rabd}{rcd} = \frac{ab}{c}$.

PROB. 6.

To reduce a mixed number to a fraction.

R U L E.

Multiply the integral part by the denomina^r, of the fraction, and to the product add the numera^r, under w^{ch} write the common denominator.

Example 1.

Let $a - \frac{b}{a}$ be given, Then $\frac{aa - b}{a}$ = Fraction required.

Example 2.

Suppose $a - x + \frac{aa - ax}{x}$, Then $\frac{ax - xx + aa - ax}{x} = \frac{aa - xx}{x}$ = Fraction req^d.

PROB. 7.

To reduce an Improper Fraction to a whole or mixed number.

R U L E.

Divide the numerator by the denomina^r, as far as you can, gives the integral part; and place the remainder over the denomina^r for the fractional part.

Example 1.

Given $\frac{ab - aa}{b}$. $b \overline{) ab - aa} \left(a - \frac{aa}{b} \right.$ Answer.
 $\frac{ab}{-aa}$

Example 2.

Suppose, $\frac{aa + xx}{a - x}$.

$a - x \overline{) aa + xx} \left(a + x + \frac{2xx}{a - x} \right.$
 $\frac{aa - ax}{ax + xx}$
 $\frac{ax - xx}{2xx}$

PROB. 8.

To find the greatest common divisor, for the terms of a fraction, or for any two quantities.

R U L E.

The quan.^s being ranged according to the dimensions of some letter; divide the greater by y^e lesser, and the last divisor by the last remainder, and so on continually till nothing remain; then the last divisor is that required. But observe, first to throw out of each divisor, all the simple divisors, (or others) that will divide it; and then proceed. The simple divisors are had by inspection.

Example 1.

Let $\frac{cd+dd}{aac+aad}$ be the fraction proposed.

$$cd+dd)aac+aad($$

$$\text{or, } c+d)aac+aad(aa$$

$$\frac{aac+aad}{0}$$

$\therefore c+d$ is the greatest common divisor.

$$c+d)\frac{cd+dd}{aac+aad}=\frac{d}{aa}.$$

Example 2.

Let $\frac{a^3-abb}{aa+2ab+bb}$ be proposed.

$$aa+2ab+bb)a^3-abb(a$$

$$\frac{a^3+2aab+abb}{-2aab-2abb)aa+2ab+bb($$

or dividing the last divisor by $-2ab$.

$$a+b)aa+2ab+bb(a+b$$

$$\frac{aa+ab}{ab+bb}$$

$$\frac{ab+bb}{0}$$

$\therefore a+b$ is the greatest common divisor.

$$\begin{array}{r} a+b \) \ a^3 - abb \ (a^2 - ab \\ \underline{a^3 + a^2b} \\ -a^2b - abb \\ \underline{-a^2b - abb} \\ 0 \end{array}$$

$\therefore \frac{a^2 - ab}{a + b} = \text{fraction in its lowest terms.}$

Example 3.

Let $\frac{a^4 - b^4}{a^5 - bba^3}$ be given.

$$\begin{array}{r} a^4 - b^4 \) \ a^5 - bba^3 \ (a \\ \underline{a^5 - b^4a} \end{array}$$

Remainder. $-bba^3 + b^4a$ $a^4 - b^4$ (

or dividing by $-bba$.

$$\begin{array}{r} a^2 - b^2 \) \ a^4 - b^4 \ (aa + bb \\ \underline{a^4 - bbaa} \\ +bbaa - b^4 \\ \underline{+bbaa - b^4} \\ 0 \end{array}$$

The common divisor is $aa - bb$.

$$\begin{array}{r} aa - bb \) \ a^5 - bba^3 \ (a^3 \\ \underline{a^5 - bba^3} \\ 0 \end{array} \quad \therefore \frac{aa + bb}{a^3} = \text{fraction in its lowest terms.}$$

PROB. 9.

To reduce a fraction to its lowest terms.

R U L E.

Find the greatest common measure (Prob. 8.) by w^{ch} divide both numerator and denom^r of the fraction; the quotients will be the numera^r and denom^r of the fraction req^d.

Example 1.

Let $\frac{cd+dd}{aac+aad}$ be proposed.

By Prob. 8. the greatest common divisor is $c+d$, $\therefore c+d \big) \frac{cd+dd}{aac+aad} = \frac{d}{aa}$ = fraction in its lowest terms.

Example 2.

Let $\frac{a^3-abb}{aa+2ab+bb}$ be proposed.

By Ex. 2. prob. 8. $a+b$ is the greatest common divisor, $\therefore a+b \big) \frac{a^3-abb}{aa+2ab+bb} = \frac{a^2-ab}{a+b} =$ = fraction in its lowest terms.

Example 3.

Suppose $\frac{a^4-b^4}{a^5-bba^3}$ to be given.

By Ex. 3. prob. 8. the greatest common divisor is $aa-bb$, $\therefore aa-bb \big) \frac{a^4-b^4}{a^5-bba^3} = \frac{a^2+b^2}{a^3}$ = fraction in its lowest terms.

PROB. 10.

To reduce fractions of different denominators, to fractions of the same value, having a common denominator.

RULE.

Multiply each numerator, into all the other denominators, for a new numerator; then xply all the denominators together for a common denominator.

Example 1.

Let $\frac{a}{b}$ and $\frac{a+b}{c}$ be given.

$$\left. \begin{array}{l} a \times c = ac \\ a+b \times b = ab+bb \end{array} \right\} \begin{array}{l} \text{New Numerators} \\ \text{common denom.} \end{array} \quad \therefore \frac{ac}{bc} \text{ and } \frac{ab+bb}{bc}; \text{ are the fractions req'd.}$$

Example 2.

Let $\frac{a}{b}$, $\frac{c}{d}$, $\frac{f}{g}$ be proposed.

$$\left. \begin{array}{l} a \times d \times g = adg \\ c \times b \times g = cbg \\ f \times b \times d = fbd \end{array} \right\} \begin{array}{l} \text{new numerators.} \\ \text{common denom.} \end{array} \quad \therefore \frac{adg}{bdg}, \frac{cbg}{bdg}, \frac{fbd}{bdg} \text{ the fractions required.}$$

Example 3.

Let $\frac{a}{2bb}$, $\frac{c}{2b}$, $\frac{d}{b}$ be given.

$$\left. \begin{array}{l} a \times 2b \times b = 2ab^2 \\ c \times 2bb \times b = 2cb^3 \\ d \times 2bb \times 2b = 4db^3 \end{array} \right\} \begin{array}{l} \text{new numerators.} \\ \text{common denom.} \end{array} \quad \text{And } \frac{2ab^2}{4b^4}, \frac{2cb^3}{4b^4}, \frac{4db^3}{4b^4}; \text{ Or,}$$

$$\frac{a}{2b^2}, \frac{cb}{2b^2}, \frac{2db}{2b^2}; \text{ all w}^{\text{ch}} \text{ fractions have a common denom.}$$

and equal to the given fractions.

Example 4.

Given, $\frac{2aa}{aa-ab}$ and $\frac{3ab-2bb}{2ac}$.

$$\begin{array}{l} 2aa \times 2ac = 4a^2c \\ 3ab-2bb \times aa-ab = 3a^3b-5b^2a^2+2ab^3 \end{array} \left. \vphantom{\begin{array}{l} 2aa \times 2ac = 4a^2c \\ 3ab-2bb \times aa-ab = 3a^3b-5b^2a^2+2ab^3 \end{array}} \right\} \text{New Numerators.}$$

$$aa-ab \times 2ac = 2a^2c-2a^2bc \text{ Common Denominator. } \therefore \frac{4a^2c}{2a^2c-2a^2bc}, \frac{3a^3b-5b^2a^2+2ab^3}{2a^2c-2a^2bc},$$

$$\text{Or } \frac{4a^2c}{2a^2c-2abc}, \frac{3a^3b-5b^2a+2b^3}{2a^2c-2abc}; \text{ are the fractions required.}$$

PROB. II.

To add fractional quantities together.

RULE.

If the fractions have not a common denominator, reduce them to one by Prob. 10. then add the numerators, and under the sum, write the common denominator.

Example 1.

Add $\frac{a}{b}$ to $\frac{c}{d}$.

$$\begin{array}{l} a \times d = ad \\ c \times b = cb \end{array} \left. \vphantom{\begin{array}{l} a \times d = ad \\ c \times b = cb \end{array}} \right\} \text{New Numerators. } \therefore \text{the fractions are become } \frac{ad}{bd}, \frac{cb}{bd} \text{ and there} \\ b \times d = bcd \text{ Common Denom.} \quad \text{Sum } \frac{ad+cb}{bd}.$$

Example 2.

Add $\frac{a}{b}, \frac{c}{d}, \frac{f}{g}$ together.

$$\begin{array}{l} a \times d \times g = adg, \quad c \times b \times g = cbg, \quad f \times b \times d = fbd \text{ New Numerators. And } b \times d \times g \\ = bdg \text{ Common Denom. } \therefore \text{the fractions are become } \frac{adg}{bdg}, \frac{cbg}{bdg}, \frac{fbd}{bdg}; \text{ and} \\ \text{there sum} = \frac{adg+cbg+fbd}{bdg}. \end{array}$$

Example 3.

Add $\frac{a-b}{3c}$ to $\frac{2b-4a+d}{3c}$.

$$\text{The sum} = \frac{a-b+2b-4a+d}{3c} = \frac{b-3a+d}{3c}.$$

Example 4.

To $a - \frac{aa}{b}$,

Add $b + \frac{a-b}{c}$.

The fractions $-\frac{aa}{b}$ and $\frac{a-b}{c}$ reduced to a Common denominator are $-\frac{aac}{bc}$ and $\frac{ba-bb}{bc}$ whose sum is $= \frac{ba-bb-aac}{bc} \therefore$ the whole sum is $=$

$$a+b + \frac{ba-bb-aac}{bc}.$$

Or thus.

$a - \frac{aa}{b} = \frac{ba-aa}{b}$, and $b + \frac{a-b}{c} = \frac{bc+a-b}{c}$. which fractions $\frac{ba-aa}{b}$ and $\frac{bc+a-b}{c}$ reduced to a Common denom^r. are $\frac{bac-aac}{bc}$ and $\frac{bbc+ba-bb}{bc}$ whose sum is $= \frac{bac-aac+bbc+ba-bb}{bc} = a+b - \frac{aac+ba-bb}{bc} = a+b + \frac{ba-bb-aac}{bc}$ as before.

PROB. 12.

To subtract one fraction from another.

RULE.

Reduce them to a Common denom^r: then subtract the numerators: and under the difference, write the Common denominator.

Example 1.

From $\frac{a+b}{d}$ subtract $\frac{c}{d}$.

$$\frac{a+b-c}{d} = \text{Difference.}$$

Example 2.

From $\frac{a+b}{d} = \frac{ab+bb}{bd}$

Subtract $\frac{aa}{b} = \frac{aad}{bd}$. then

$$\frac{ab+bb-aad}{bd} = \text{difference.}$$

Example 3.

From $\frac{a-b}{3c}$ take $\frac{2b-4a}{5d}$.

Reduced $\frac{5ad-5bd}{15cd}$; $\frac{6bc-12ac}{15cd}$.

$$\text{difference. } \frac{5ad-5bd-6bc+12ac}{15cd}.$$

Example 1.

$$\text{From } a - \frac{aa}{b} \text{ or } a - \frac{aac}{bc}$$

$$\text{Take } b + \frac{a-b}{c} \text{ or } b + \frac{ab-bb}{bc}$$

$$\text{Difference. } a-b + \frac{bb-ab-aac}{bc}.$$

PROB. 13.

To Multiply Fractions.

RULE.

Multiply the Numerators together for a new Numerator; and the Denom^{rs} together for a new denominator.

$$\text{Multiply } \frac{a}{b} \text{ by } \frac{c}{d}$$

$$\text{then, } \frac{a \times c}{b \times d} \text{ or } \frac{ac}{bd} = \text{product.}$$

Example 2.

$$\text{Multiply } \frac{b}{c} \text{ by } \frac{a+b}{b+c}.$$

$$\frac{b}{c} \times \frac{a+b}{b+c} = \frac{ab+bb}{bc+cc} \cdot \text{product.}$$

Example 3.

Multiply $\frac{aa-bb}{bc}$ by $\frac{aa+bb}{b+c}$.

then, $\frac{aa-bb}{bc} \times \frac{aa+bb}{b+c} = \frac{a^4-b^4}{bbc+bcc} = \text{Product.}$

For the sum of two quantities $\times$ d by their difference, gives the difference of their squares.

For let $a+b$ be $\times$ d $a-b$.

$$\begin{array}{r} a+b \\ a-b \\ \hline aa+ab \\ -ab-bb \\ \hline \end{array}$$

Then, $\frac{aa-bb}{\quad} = \text{Product.}$

Example 4.

Multiply $\frac{aabb}{3cdd}$ by $\frac{c}{ad}$.

$\frac{bb}{3dd} = \text{Product.}$

Example 5.

Multiply $\frac{aa+2ab+bb}{cd-dd}$ by $\frac{dd}{a+b}$.

Then, $\frac{ad+bd}{c-d} = \text{Product.}$

Example 6.

Multiply $\frac{aa-2bb}{a-b}$ by $a-b$

then, $aa-2bb = \text{product.}$

Example 7.

Multiply $\frac{aa+3bb}{3cd}$ by xx .

Product $\frac{aaxx+3bbxx}{3cd}$.

Example 8.

Multiply $\frac{2a-2x}{3b}$ by $a+x$.

Then, $\frac{2aa-2ax+2ax-2x}{3b} = \frac{2aa-2xx}{3b} = \text{product.}$

PROB. 14.

To divide one fraction by another.

RULE.

*Multiply the denom^r of the divisor by the numerator of the dividend, for a new Numerat^r.
and the numerat^r of the divisor into the denom^r of the dividend, for a new denomina^r.*

Example 1.

Divide $\frac{a}{b}$ by $\frac{c}{d}$.

$\frac{c}{d}) \frac{a}{b} (\frac{ad}{bc} = \text{Quot.}$

Example 2.

Divide $\frac{a+b}{a}$ by $\frac{a+c}{a-c}$.

$$\frac{a+c}{a-c} \Big) \frac{a+b}{a} \left(\frac{aa+ba-ac-bc}{aa+ac} = \text{Quote.} \right.$$

Example 3.

Divide $\frac{a+b}{a}$ by $\frac{a+c}{a-b}$.

$$\frac{a+c}{a-b} \Big) \frac{a+b}{a} \left(\frac{aa-bb}{aa+ac} = \text{Quote.} \right.$$

Example 4.

Divide $\frac{aa-bb}{a+d}$ by $\frac{2ab-bb}{a+d}$.

$$\frac{aa-bb}{2ab-bb} \text{ quotient.}$$

Example 5.

Let $\frac{aa+2ab+bb}{c-d}$ divide $\frac{a^3-ab^2}{c-d}$.

$$\frac{a^3-ab^2}{aa+2ab+bb} = \frac{aa-bb \times a}{aa+2ab+bb} = \frac{\overline{a+b} \times \overline{a-b} \times \overline{a}}{\overline{a+b} \times \overline{a+b}} = \frac{aa-ba}{a+b} = \text{quote.}$$

Example 6.

Divide $\frac{a-b}{c}$ by d .

$$\frac{d}{1} \Big) \frac{a-b}{c} \left(\frac{a-b}{dc}, \text{ quote.} \right.$$

Example 7.

Let $a+b$ divide $\frac{aa-2bb}{a-b}$.

$$\frac{a+b}{1} \Big) \frac{aa-2bb}{a-b} \left(\frac{aa-2bb}{aa-bb} = \text{quote.} \right.$$

Example 8.

Let $\frac{a-b}{cd}$ divide $\frac{aa-bb}{c+d}$.

$$\text{Quote} = \frac{\overline{aa-bb} \times \overline{cd}}{\overline{a-b} \times \overline{c+d}} = \frac{\overline{a+b} \times \overline{a-b} \times \overline{cd}}{\overline{a-b} \times \overline{c+d}} = \frac{\overline{a+b} \times \overline{cd}}{\overline{c+d}} = \frac{acd+bcd}{c+d}.$$

Example 9.

Let $\frac{aa-ab}{a-b}$ divide $\frac{a^4-b^4}{aa-2ab+bb}$.

$$\text{Divisor} = \frac{\overline{a-b} \times \overline{a}}{\overline{a-b}} = \overline{a}. \quad \text{Dividend} = \frac{\overline{a^2+b^2} \times \overline{a^2-b^2}}{\overline{a-b} \times \overline{a-b}} = \frac{\overline{a^2+b^2} \times \overline{a+b} \times \overline{a-b}}{\overline{a-b} \times \overline{a-b}} =$$

$$= \frac{\overline{a^2+b^2} \times \overline{a+b}}{\overline{a-b}} = \frac{a^3+ab^2+aab+b^3}{a-b}$$

And,

$$\frac{a}{1} \Big) \frac{a^3+ab^2+aab+b^3}{a-b} \left(\frac{a^3+ab^2+aab+b^3}{aa-ab} = \text{quote.} \right.$$

Scholium. 1.

The value of any fractional quantity is not at all changed, by changing all the signs of both numerator and denominator.

Thus $\frac{ab-ac}{r-c} = \frac{ac-ab}{c-r}$. For, Put $\frac{ab-ac}{r-c} = n$, then $\frac{ab-ac \times r-c}{r-c} = nr-nc$
 $\therefore ab-ac = nr-nc$ and $ac-ab = nc-nr \therefore \frac{ac-ab}{c-r} = n$ as before.

2.

The value of any compound fractional quantity is equal to the sum of all the particular simple fractions, that compose it.

Thus, $\frac{rx + 2cx - 11rz}{3r-2x}$ is $= \frac{rx}{3r-2x} + \frac{2cx}{3r-2x} - \frac{11rz}{3r-2x}$. For by addition of fractions the sum of $\frac{rx}{3r-2x} + \frac{2cx}{3r-2x} = \frac{rx+2cx}{3r-2x}$; and by subtraction $\frac{11rz}{3r-2x}$ taken from this sum, makes $\frac{rx+2cx-11rz}{3r-2x}$ as before.

3.

If a fraction be xed by any given quantity; it is the same thing whether the numerator be xed by that quantity, or the denominator divided by it.

$$\text{For } \frac{dab}{dc} \times d = \frac{dabd}{dc} = \frac{dab}{c}.$$

4.

If a fraction is to be divided by some quantity; it is the same thing whether the numerator be divided by it, or the denominator xed by it.

$$\text{For, } \frac{2az}{x} \div r = \frac{2az}{rx}, \text{ And, } \frac{2ar}{x} \div r = \frac{2ar}{rx} = \frac{2a}{x}.$$

5.

If any sort of quantity is to be divided by a fraction; it is the same thing, as to multiply the said quantity, by the fraction inverted.

Thus, $ab \div \frac{r}{s} = ab \times \frac{s}{r}$. And $\frac{a}{c} \div \frac{b}{r}$ or $\frac{\frac{a}{c}}{\frac{b}{r}} = \frac{a}{c} \times \frac{r}{b} = \frac{ar}{bc}$.

PROB. 15.

To involve a quantity to any power.

RULE 1.

Multiply the quantity so often into itself as the index denotes. And where the root is +, all the powers are +. And where the root is -, all the odd powers are -, and all the even powers +.

Example 1.

a root.	a^2 root.	$-2a^3$ root.
aa square.	a^4 square.	$+4a^6$ square.
a^3 cube.	a^6 cube.	$-8a^9$ cube.
a^4 4 th power.	a^8 4 th power.	$+16a^{12}$ 4 th power.
¶c.	¶c.	¶c.

Example 2.

ab root.	$-3abb$ root.	$-10b^2c^3$ root.
$aabb$ square.	$+9aabb^4$ square.	$+100b^6c^6$ square.
a^3b^3 cube.	$-27a^3b^6$ cube.	$-1000b^6c^9$ cube.
¶c.	¶c.	¶c.

Example 3.

Involve $a+b$ to the 6th power.

Root.

$$\begin{array}{r} a+b \\ a+b \\ \hline aa+ab \\ ab+bb \end{array}$$

Square.

$$\begin{array}{r} aa+2ab+bb \\ a+b \\ \hline a^3+2aab+abb \\ aab+2abb+b^3 \end{array}$$

Cube.

$$\begin{array}{r} a^3+3aab+3abb+b^3 \\ a+b \\ \hline a^4+3a^3b+3a^2b^2+ab^3 \\ a^3b+3a^2b^2+3ab^3+b^4 \end{array}$$

4th power.

$$\begin{array}{r} a^4+4a^3b+6a^2b^2+4ab^3+b^4 \\ a+b \\ \hline a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4 \\ a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5 \end{array}$$

5th power.

$$\begin{array}{r} a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5 \\ a+b \\ \hline a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5 \\ a^5b+5a^4b^2+10a^3b^3+10a^2b^4+5ab^5+b^6 \end{array}$$

6th power.

$$\begin{array}{r} a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6 \end{array}$$

RULE 2.

In a binomial. The power will consist of 1 term more than the index of the power. The highest power of both is the index of the given power, and the index of the leading quantity continually decreases by 1 in every term, and in the following quantity, the indices of the terms are 0, 1, 2, 3, 4, &c. Then for finding the unciae or coefficients.

The first is always 1; the second, the index of the

Example 4.

Involve $a-b$ to the 6th power.

Root.

$$a-b$$

$$a-b$$

$$aa-ab$$

$$-ab+bb$$

Square.

$$aa-2ab+bb$$

$$a-b$$

$$a^3-2a^2b+ab^2$$

$$a^2b+2ab^2-b^3$$

Cube.

$$a^3-3a^2b+3ab^2-b^3$$

$$a-b$$

$$a^4-3a^3b+3a^2b^2-ab^3$$

$$-a^3b+3a^2b^2-3ab^3+b^4$$

4th power.

$$a^4-4a^3b+6a^2b^2-4ab^3+b^4$$

$$a-b$$

$$a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4$$

$$-a^4b+4a^3b^2-6a^2b^3+4ab^4-b^5$$

5th power.

$$a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5$$

$$a-b$$

$$a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5$$

$$-a^5b+5a^4b^2-10a^3b^3+10a^2b^4-5ab^5+b^6$$

6th power.

$$a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6$$

&c.

power. And in General, if the coefficient of any term be divided by the index of the leading quantity, and divided by the number of terms to that place; it gives the Coefficient of the next following term.

Lastly, when both terms of the root are +, all the terms of the power will be +; but if the second term be -; then all the odd terms will be +, and all the even terms -.

Example 5.

Involve $a+e$ to the 5th power.

By Rule 2^d the several terms without the coefficients will be.

$$a^5, \quad ae^4, \quad a^2e^3, \quad ae^4, \quad e^5; \text{ and the}$$

$$\text{Coefficients } 1, \quad 5, \quad \frac{5 \times 4}{2}, \quad \frac{10 \times 3}{3}, \quad \frac{10 \times 2}{4}, \quad \frac{5 \times 1}{5};$$

$$\text{that is } 1, \quad 5, \quad 10, \quad 10, \quad 5, \quad 1; \text{ and } \therefore \text{ the}$$

$$5\text{th power } a^5 + 5ae^4 + 10a^2e^3 + 10a^2e^3 + 5ae^4 + e^5.$$

Example 6.

Involve $a-x$ to the 4th power.

$$\text{By Rule the Second the power is, } a^4 - 4ax^3 + \frac{4 \times 3}{2} a^2x^2 - \frac{6 \times 2}{3} ax^3 + \frac{4 \times 1}{4} x^4.$$

$$\text{That is } a^4 - 4ax^3 + 6a^2x^2 - 4ax^3 + x^4.$$

RULE 3.

In Trinomials, Quadrinomials, &c. Let one letter remain, and put another letter for the rest of the quantities; then involve this binomial by Rule 2, then instead of the powers of the assumed letter, find by rule 2. the powers of the compound quantity it represents, which put in its stead.

Example 7.

Involve $a+b-x$ to the third power.

Put $b-x=e$, then (p. rule 2.) $\overline{a+e}^3 = a^3 + 3ade + 3aee + e^3$, that is,

$$a^3 + 3aax\overline{b-x} + 3ax\overline{b-x}^2 + \overline{b-x}^3. \text{ But}$$

by rule 2.

$$\overline{b-x}^2 = b^2 - 2bx + x^2.$$

$$\overline{b-x}^3 = b^3 - 3b^2x + 3bx^2 - x^3. \quad \therefore$$

$$\overline{a+b-x}^3 = a^3 + 3aab - 3aax + 3ab^2 - 6abx + 3ax^2 + b^3 - 3b^2x + 3bx^2 - x^3.$$

Corollary 1.

The n^{th} power of $a+e$, that is,

$$\overline{a+e}^n = a^n + na^{n-1}e + n \cdot \frac{n-1}{2} a^{n-2}e^2 + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} a^{n-3}e^3 + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} a^{n-4}e^4 + a \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot \frac{n-3}{4} \cdot \frac{n-4}{5} a^{n-5}e^5, + \&c.$$

This rule is proved by involving $a+e$ as far as you please, for the several powers will always agree with the rule.

Let it be required to raise $a+x$ to the 4th power by this rule. here $n=4$.

$$\therefore \overline{a+x}^4 = a^4 + 4a^3e + 4 \cdot \frac{4-1}{2} a^{4-2}e^2 + 4 \cdot \frac{4-1}{2} \cdot \frac{4-2}{3} a^{4-3}e^3 + 4 \cdot \frac{4-1}{2} \cdot \frac{4-2}{3} \cdot \frac{4-3}{4} a^{4-4}e^4. \text{ Or}$$

$$\overline{a-x}^4 = a^4 + 4a^3e + 4 \cdot \frac{3}{2} a^2e^2 + 4 \cdot \frac{3}{2} \cdot \frac{2}{3} ae^3 + 4 \cdot \frac{3}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} a^0e^4.$$

$$\text{Or, } \overline{a-x}^4 = a^4 + 4a^3e + 6a^2e^2 + 4ae^3 + e^4.$$

N.B. $a^0 = 1$, as will be seen hereafter.

RULE 4.

Multiply the index of the quantity, by the index of the power, and make the signs as in rule 1.

Example 8.

Root.	a'	$-2bba$, or $-2b^2a$
Square.	$\overline{a}^{1 \times 2} = a^2$	$-\overline{2b^2a}^2 = +4 \times \overline{b}^{2 \times 2} \times \overline{a}^{1 \times 2} = +4b^4a^2$
Cube.	$\overline{a}^{1 \times 3} = a^3$	$-\overline{2b^2a}^3 = -8 \times \overline{b}^{2 \times 3} \times \overline{a}^{1 \times 3} = -4b^6a^3$
mth power.	$\overline{a}^{1 \times m} = a^m$	$-\overline{2b^2a}^m = -2^m \times \overline{b}^{2 \times m} \times \overline{a}^m = -2^m \times \overline{b}^{2m} \times \overline{a}^m$

Example 9.

Root.	$\overline{a-x}^2$
Square.	$\overline{a-x}^{2 \times 2} = \overline{a-x}^4$
Cube.	$\overline{a-x}^{2 \times 3} = \overline{a-x}^6$
mth power.	$\overline{a-x}^{2 \times m} = \overline{a-x}^{2m}$

October 16. 1776.

PROB. 16.

To extract the root of any quantity.

RULE 1.

For simple quantities; extract the root of the Coefficient for the numerical part, and $\div$ d the index of the letter or letters, by the index of the power, gives the index of the root.

Example 1.

The Cube root of a^3 is $a^{\frac{3}{3}}$ or a .

The Square root of $25a^4$ is $5a^{\frac{4}{2}}$ or $5aa$.

The square root of $2a^2b^6$ is $a^{\frac{2}{2}} \times b^{\frac{6}{2}} \times \sqrt{2} = ab^3\sqrt{2}$.

The Cube root of $-125b^9$ is $-5b^{\frac{9}{3}} = -5b^3$.

RULE 2.

For the square root, of a compound quantity; range the terms according to the dimensions of some letter. Then find the root of the first term by rule 1, and set in the quotient: subtract its square, and bring down the next term, which divide by double the quotient, and set the answer in the quotient. Multiply the divisor and quotient by this last quotient, which subtract from the dividend, proceed thus, just as in Common Arithmetic.

Example 2.

Extract the square root of $aa+2ab+bb$.

$$\begin{array}{r} aa+2ab+bb \quad (a+b. \\ aa \\ \hline 2a+b) \overline{2ab+bb} \\ \quad 2ab+bb \\ \hline \quad \quad 0 \end{array}$$

Example 3.

Extract the square root of $aa + 4ab + 4bb - 2ax - 4bx + xx$.

$$\begin{array}{r}
 aa + 4ab + 4bb - 2ax - 4bx + xx \\
 \underline{aa} \\
 2a+2b \big) 4ab + 4bb \\
 \underline{4ab + 4bb} \\
 2a+2b-x \big) -2ax - 4bx + xx \\
 \underline{-2ax - 4bx + xx} \\
 0
 \end{array}
 \quad
 \begin{array}{l}
 (a+2b-x \text{ Root.} \\
 \underline{a+2b-x} \\
 aa+2ba-ax \\
 +2ba+4bb-2bx \\
 -ax-2bx+xx \\
 \hline
 aa+4ba+4bb-2ax-4bx+xx
 \end{array}$$

Example 4.

Extract the square root of $aa - 6na + 9nn + 2za - 6nz + zz$.

$$\begin{array}{r}
 aa - 6na + 9nn + 2za - 6nz + zz \\
 \underline{aa} \\
 2a-3n \big) -6na + 9nn \\
 \underline{-6na + 9nn} \\
 2a-6n+z \big) 2za - 6nz + zz \\
 \underline{2za - 6nz + zz} \\
 0
 \end{array}
 \quad
 \begin{array}{l}
 (a-3n+z. \text{ Root.} \\
 \underline{a-3n+z} \\
 aa-3na+a^2 \\
 -3na+9nn-3nz \\
 +az-3nz+zz \\
 \hline
 aa-6na+9nn+2az-6nz+zz
 \end{array}$$

Example 5.

Extract the square root of $a^4 - 2a^2x + 3a^2x^2 - 2ax^3 + x^4$.

$$\begin{array}{r}
 a^4 - 2a^2x + 3a^2x^2 - 2ax^3 + x^4 \\
 \underline{a^4} \\
 2a^2-ax \big) -2a^2x + 3a^2x^2 \\
 \underline{-2a^2x + 3a^2x^2} \\
 2a^2-ax+x^2 \big) 2a^2x^2 - 2ax^3 + x^4 \\
 \underline{2a^2x^2 - 2ax^3 + x^4} \\
 0
 \end{array}
 \quad
 \begin{array}{l}
 (a^2-ax+x^2 \text{ Root.} \\
 \underline{a^2-ax+x^2} \\
 a^4-a^2x+a^2x^2 \\
 -a^2x+a^2x^2-ax^3 \\
 +a^2x^2-ax^3+x^4 \\
 \hline
 a^4-2a^2x+3a^2x^2-2ax^3+x^4
 \end{array}$$

Example 6.

Extract the square root of $a^2 + x^2$.

$aa + xx \left(a + \frac{xx}{2a} - \frac{x^4}{8a^3} + \frac{x^6}{16a^5} - \text{&c.} \right)$ Root nearly.

$$\begin{array}{r}
 aa \\
 2a + \frac{xx}{2a} \bigg) \quad + xx \\
 \quad + xx + \frac{x^4}{4a^2} \\
 \hline
 2a + \frac{xx}{a} - \frac{x^4}{8a^3} \bigg) \quad - \frac{x^4}{4a^2} \\
 \quad - \frac{x^4}{4a^2} - \frac{x^6}{8a^4} + \frac{x^8}{64a^6} \\
 \hline
 2a + \frac{xx}{a} - \frac{x^4}{4a^3} + \frac{x^6}{16a^5} \bigg) \quad + \frac{x^6}{8a^4} - \frac{x^8}{64a^6} \\
 \quad + \frac{x^6}{8a^4} + \frac{x^8}{16a^6} - \frac{x^{10}}{64a^8} + \frac{x^{12}}{256a^{10}} \\
 \hline
 \quad - \frac{5x^8}{64a^6} + \frac{x^{10}}{64a^8} - \frac{x^{12}}{256a^{10}} \text{ Remainder.}
 \end{array}$$

Thus you may go on as far as you please, continuing the root in an infinite series, which when x is small in respect to a , a few of the first terms will give the root very near the truth; and therefore all the other terms, which are but very small in respect to these, may be safely rejected; as will appear thus.

Suppose $x = 2$ and $a = 10$, then $\sqrt{aa + xx} = \sqrt{104} = 10,19805$.

$$\begin{array}{r}
 a = 10 \\
 \frac{x^2}{2a} = 0,2 \\
 \hline
 a + \frac{x^2}{2a} = 10,2 \\
 - \frac{x^4}{8a^3} = 0,002 \\
 \hline
 10,198 = a + \frac{xx}{2a} - \frac{x^4}{8a^3}
 \end{array}$$

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RULE 3.

1.

For the Cube root of a compound quantity, range the terms according to the dimensions of some letter. Then find the root of the first term and set it in the quotient: cube this root and subtract it from the compound quantity for the resolvend.

2.

For the divisor, triple the square of the root for the first part of the divisor, and triple the same root for the second part of the divisor. And every following divisor, must be triple the square of the sum of all the simple quantities, or parts of the root already found out and set in the quotient, and of the said sum.

3.

Divide only the first simple quantity in the resolvend, by the first simple quantity in the divisor, and set that which comes forth in the quotient next after the term or terms of the root before found with its proper sign.

4.

After the first simple Cube is subtracted, by the first part of this rule, then every following Subtrahend, or sum of the quantities to be subtracted from the Resolvend, must be composed of these products.

N^o 2. The first part of the divisor $\times d$ by the last quantity found in the quotient. added to the product of the latter part of the divisor by the square of the same simple quantity. And the Cube of the said simple quantity.

Example 7.

Extract the Cube root of $aaa + 3aae + 3aee + eee$.

$$\begin{array}{r} aaa + 3aae + 3aee + eee \quad (a + e \\ \underline{aaa} \end{array}$$

$$\underline{3aae + 3aee + eee} \text{ Resolvend.}$$

$$\underline{3aa + 3a} = aa \times 3 + a \times 3 = \text{divisor.}$$

$$\text{Subtrahend.} \quad \underline{3aae + 3aee + eee = 3aa \times e + 3a \times ee + eee.}$$

$$0$$

Example 8.

Extract the Cube root of $125a^3 + 225a^2e + 135ae^2 + 27e^3$.

$$\begin{array}{r} 125a^3 + 225a^2e + 135ae^2 + 27e^3 \quad (5a + 3e. \\ \underline{125a^3} \end{array}$$

$$\underline{225a^2e + 135ae^2 + 27e^3} \text{ Resolvend.}$$

$$\underline{75a^2 + 15a} = \text{Divisor} = 5a \sqrt{3} + 5a \times 3.$$

$$\text{Subtrahend} \quad \underline{225a^2e + 135ae^2 + 27e^3 = 75a^2 \times 3e + 15a \times 3e^2 + e^3}$$

$$0$$

Example 9.

Extract the Cube root of $a^3 - 3a^2e + 3ae^2 - e^3$ ($a - e$).

$$\begin{array}{r} a^3 - 3a^2e + 3ae^2 - e^3 \text{ Resolvend.} \\ \underline{a^3} \end{array}$$

$$\underline{3a^2 + 3a} \text{ divisor.}$$

$$\underline{-3a^2e + 3ae^2 - e^3} \text{ Subtrahend.}$$

$$0$$

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